Low energy ion beam propagation in own gas

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Low energy quasineutral ion beam propagation in own gas is considered. Criterion of background gas concentration uniformity is obtained. It is shown that the generation of slow ions occurs preferentially due to process of a resonant charge exchange. The potential of a quasineutral ion beam with respect to the walls of the vacuum chamber is defined. The limiting current of quasineutral ion beam in the region of ion drift is obtained. The limiting current is proportional to concentration of residual gas, to radius of drift tube, to a cube of an accelerating voltage and inversely proportional to potential of ion-beam plasma raised to the power of one and a half.

1. Introduction

Electric ion propulsion thrusters are widely used for correction of orbits of long-life spacecrafts and for controlling their orientation in space today. The most popular type of such thrusters are Hall thrusters, in which the gas discharge in the crossed electrical and magnetic fields is used. Working gas is delivered into the discharge channel of the thruster, where it is exposed to ionization depletion under activity of Hall electrons [1].

The ions, formed during ionization depletion, are accelerated by electric field. The low energy ion beam propagates in own gas after an output of Hall thruster. It is necessary to distinguish operating modes of electric propulsion thrusters in laboratory conditions and in conditions of outer space. In laboratory conditions thruster works in the vacuum chamber filled with working gas. In space conditions the stream of neutral gas is also injected alongside with a stream of the accelerated ions into outer space but only when ionization of working gas in Hall thruster is incomplete.

Examining operation of the Hall thruster prototype in laboratory conditions we deal with a beam of high velocity ions with a current of the order of one ampere and a not large amount of kinetic energy (100–300 eV), propagated in some drift tube filled with own gas. The positive space charge of this beam with necessity should be compensated by the negative charge of slow Maxwellian electrons. The basic process of interaction of high velocity ions with their own gas in a considered range of energies is process of a resonant charge exchange. As a result of this process high velocity ion is turned into slow ion, so that an effective deceleration of beam ions takes place there. The electrons compensating the positive charge of a beam can be generated by three ways: firstly, due to secondary ion-electron emission from walls of drift tube; secondly, by ionization of gas by thermal electrons; and, thirdly, by means of some exterior source of electrons often called the cathode-compensator. Thus, we deal with rather special case of ion-beam plasma, which we will call “charge exchange plasma”.

In this report we shall not consider questions related to presence of the cathode-compensator. It is obvious, that at low temperatures of electron gas process of volume ionization in ion-beam plasma is not effective. For this reason there is some electron temperature below which it is possible to neglect volume ionization by thermal electrons. Supposing that slow ions in volume of a drift tube are generated exclusively due to process of a resonant charge exchange of high velocity ions of a beam, we can estimate value of this characteristic temperature.

2. “Charge exchange plasma”

Supposing that concentration of neutral particles is constant, we consider process of the ion beam propagation in the region of ion drift. Let us write down the equations of continuity for high velocity and slow ions that take into account radial departure of slow ions and process of a resonant charge exchange. Really, if we consider that high velocity ions and neutral particles move parallel to axis $x$, then we can write down the equations of continuity in the following form
\[ dJ_i(x)/dx = -J_i(x)n_e \sigma_n + eQ(x)n_i(x)\sigma_n, \]  
(1)

\[ 2\pi en_i(x) v = J_i(x)n_e \sigma_n - eQ(x)n_i(x)\sigma_n, \]  
(2)

where \( J_i(x) \) – current of the ion beam; \( n_e \) – concentration of working gas in the drift tube; \( \sigma_n \) – cross-section of process of a resonant charge exchange; \( Q(x) \) – flux of high velocity neutral particles; \( n_i(x) \) – concentration of slow ions; \( a \) – radius of ion beam; \( e \) – absolute value of the elementary electrical charge; \( v_s = \sqrt{T_e/M} \) – ion-sound velocity; \( T_e \) – plasma electrons temperature measured in energetic units; \( M \) – mass of ion. As boundary conditions for this equations set we take

\[ J_i(0) = J_{i0}; \quad Q(0) = 0. \]  
(3)

From the equation (2) follows the presence of the some “charge exchanging” scale of a current [2]

\[ J_i = 2\pi nev_i/\sigma_n. \]  
(4)

Thus, for the xenon ion beam, ions of which have energies of the order of 100÷300 eV, \( \sigma_n \approx 5 \cdot 10^{-15} \) cm² and

\[ J_i \approx 17.3a/\sqrt{T_e}, \]  
where radius of the ion beam is expressed in centimeters, temperature of electrons in electron-volts and ion current in amperes.

It is easy to show that if the inequality

\[ J_{i0} << J_i, \]  
(5)
takes place then we can neglect second terms in equations (1) and (2)\(^1\). Therefore in considered conditions we can write down

\[ J_i(x) = J_{i0} \exp(-n_e \sigma_n x) = J_{i0} \exp(-x/\lambda), \]  
(6)

where \( \lambda \) – charge exchange length.

If we assume that slow ions are generated only as a result of process of a resonant charge exchanging in the region of beam, then radial component of density of a current of slow ions outside of a beam is

\[ j_i(r,x) = J_i(x)/2\pi r \lambda. \]  
(7)

To estimate the rate of gas ionization by thermal electrons we take [3]

\[ \frac{dn_i}{dt} \approx n_n n_e \sigma_m \langle v_s \rangle(1 + 2T_e/\exp(\varphi_i/T_e)) \exp(-\varphi_i/T_e)/4, \]  
(8)

where \( n_n \approx n_i \) – concentration of ion-beam plasma; \( \langle v_s \rangle = \sqrt{8T_e/\pi m} \) – average absolute value of thermal electron velocity; \( m \) – mass of electron; \( \varphi_i \) – working gas ionization potential. Assuming that the stream of the slow ions, generated due to a charge exchange process of beam high velocity ions, on a wall of a drift tube overcomes 10 times a stream of the slow ions, generated due to ionization of gas by thermal electrons, we shall derive

\[ \sqrt{5n_e \sigma_m R M/M(1 + 2T_e/\exp(\varphi_i/T_e)) \exp(-\varphi_i/T_e)} < 1. \]

It is easy to show that in typical for testing Hall thrusters conditions (working gas is xenon, the flow rate of xenon is of the order of 1 amper), energy of ions is approximately equal 100÷300 eV, radius of vacuum chamber is of the order of 25 cm) the characteristic electronic temperature will be equal 2÷4 eV.

3. Equilibrium plasma potential

The potential of plasma is defined by balance of an electron component. If secondary ion-electron emission from walls of a drift tube (\( \gamma \) – coefficient of secondary ion-electron emission [3]) is the only source of electrons in a volume of ion-beam plasma then there is dynamic balance between current of secondary electrons and chaotic current of electrons from plasma that have overcome potential barrier \( \varphi_{pl} \):

\[
2\pi R j_i(R,x)dx = 2\pi R(2\sqrt{2\pi})^{-1}\sqrt{M/m} \times 
\exp\left(-\varphi_{pl}/T_e\right)\int_{0}^{\infty}j_i(R,x)dx. 
\]  
(9)

Taking into consideration (6) and (7), from (9) for potential of ion-beam plasma with respect to the walls of drift tube we obtain [2]

\[ \varphi_{pl} = T_e \ln\left(2\gamma\sqrt{2\pi}^{-1}\sqrt{M/m}\right). \]  
(10)

Electron temperature is defined by equation of balance of energy for electron gas. In integral form it can be written as:

\[
2\pi R \left(\varphi_{pl} + e_T\right)\int_{0}^{\infty}j_i(R,x)dx = 2\pi R(\varphi_{pl} + 2T_e)(2\sqrt{2\pi})^{-1}\sqrt{M/m} \times 
\exp\left(-\varphi_{pl}/T_e\right)\int_{0}^{\infty}j_i(R,x)dx. 
\]  
(11)

\(^1\) Let us note that assumption about uniformity of working gas concentration (\( n_e = const \)) is also connected with condition of smallness of ion current (5). However, in this case condition of smallness of current of slow ions from the region of the beam in comparison with chaotic flux of neutral gas leads to a rather stronger inequality \( J_{i0}/J_i << \sqrt{T_e/T_e} = 0.1 \). where \( T_e \) – temperature of gas in a drift tube.
where $E_\gamma$ – average energy of $\gamma$-electrons. Taking into account (6), (7) and (10) from (11) we obtain

$$T_\epsilon = \frac{E_\gamma}{2},$$

(12)

namely in typical conditions it will be equal 1.5-2 eV.

4. Limiting ion beam current

The system reacts to external perturbation (random or caused by external action) by some change of its parameters. Instability arises if between the external perturbation and the response of system to this perturbation there is positive regenerative feedback or synchronism. Finding conditions of occurrence of such synchronism is the first priority interest for stability analysis of system.

Let us consider problem of limiting current of cylindrical quasineutral ion beam from this point of view. This problem was considered by Yu.S. Popov [4] for one-dimensional case and by A.V. Zharinov [5] for ribbon ion beam. Let ion beam, radially homogeneous, with radius equal to $a$, flow in a drift tube with radius $R$. Current of a beam at a plane of injection $(x=0)$ is equal to $J_{i0} = \pi a^2 \rho_{i0}$, where $u_i = \sqrt{2e\varphi_0/M}$ is velocity of ions that have been accelerated by a potential difference of $\varphi_0$. While propagating along axis $x$ current of the beam decreases exponentially according to equation (6).

At equilibrium state space charge of ion beam is neutralized by electrons. Velocity distribution of these electrons can be assumed as Maxwellian. All volume of drift tube will be filled with plasma which has potential with respect to the walls of drift tube at an equilibrium state equal to $\varphi_{pl}$.

Let us suppose that random fluctuation of potential, $\delta\varphi_1$, that decelerates ions, occurs in the beam. It will lead to decreasing velocities of ions by the value $\delta u_i$ ($\delta u_i = -u_i \delta\varphi_1 / 2\varphi_0$). Assuming that density of current of the ion beam is constant for total excess positive charge of the beam ($e\delta N_{ib}$) we can obtain

$$e\delta N_{ib} = \lambda \delta\varphi_1 J_{i0} / 2\varphi_0 u_i.$$  

(13)

Almost instantly, during the time interval of the order of $R/c$, this charge will be neutralized by Maxwellian electrons, plasma will remain quasineutral and this excess of positive charge will appear in near-wall layers. Ion-beam plasma reacts to bringing into it an excess positive charge similarly to a metal conductor - it pushes out a charge on a surface.

Let us continue this comparison with a conductor and imagine ion-beam plasma in a drift tube as a cylindrical capacitor with capacity equal to $C$. Then if we put excess charge $e\delta N_{ib}$ on the inner conductor of capacitor it will lead to increase of the voltage of capacitor by the value $\delta\varphi = e\delta N_{ib} / (C + \varphi_{pl} dC / d\varphi_{pl})$. If this value exceeds initial fluctuation of potential ($\delta\varphi_1$) then this ion beam will be unstable.

Taking into account Child-Langmuir law for the current of slow ions to the wall of drift tube for the capacity of this cylindrical capacitor we can write down

$$C(\varphi_{pl}) \approx (2R/3) \int_0^\infty dx / d(x) =$$

$$= 4R \lambda \sqrt{J_{i0} (2R\lambda \varphi_{pl}^{3/2})^{-1} \sqrt{M / 2e}},$$

(14)

where $d(x)$ – thickness of the space charge sheath.

Thus, limiting current of quasineutral ion beam in region of drift is equal to

$$J_{max} = 2(R/\lambda)(\varphi_0 / \varphi_{pl})^{3/2} \sqrt{2e/M \varphi_{pl}^{3/2}}.$$  

(15)

Limiting current is proportional to the concentration of residual gas, to the radius of drift tube, to the cube of accelerating voltage and is inversely proportional to the potential of the ion-beam plasma raised to power of one and a half.

In figure 1 dependences of limiting current of the quasineutral xenon ion beams on working voltage of the Hall thruster are shown for different values of potential of ion-beam plasma ($\varphi_0 >> \varphi_{pl}$). It was assumed that ratio $R/\lambda = 10$. As an example flow rate of xenon for Hall thruster SPT-100 expressed in current units ($Q = 3.91$ A) is marked by horizontal
dashed line. Vertical dashed line corresponds to working voltage of SPT-100 ($\varphi_0 = 293.3$ V). Thus, circle in figure 1 corresponds to an operating point of SPT-100 on the assumption that depletion of working gas is total.

It is obvious that it is necessary for limiting current of electrostatic instability to exceed flow rate of working gas for normal Hall thruster operation. We can see in figure 1 that if potential of ion-beam plasma is large enough then ion beam of SPT-100 in the drift tube with given conditions becomes unstable.

Estimations given above show that the ion beam in laboratory operating conditions of Hall thruster can easily become unstable against formation of “virtual anode”. There are at least two reasons leading to an underrating of estimated value of a critical current of electrostatic instability of quasineutral ion beam. Firstly, in this report we did not consider influence of the cathode-compensator at all. Moreover we did not consider compensating influence of secondary ion-electron emission at a derivation of a relation (15). Thus, we have simplified a problem, but have passed over an ability of autoneutralization of the ion beam. Secondly, averaging procedure on longitudinal coordinate, namely smoothing of longitudinal inhomogeneities of system, was used at a derivation of the equation (15). Such procedure always leads to underrating of the instability threshold because it actually substitutes for real nonuniform system (in which synchronism between external perturbation and the response of system has local character) some homogeneous system (in which synchronism has global character).

5. Conclusion

In the this report stability of the ion beam with a current of the order of one ampere and a small amount of kinetic energy (100÷300 eV) in region of drift is considered. First of all the problem of the description of an equilibrium state of quasineutral ion beam in the drift tube filled with own gas is considered. In particular the potential of quasineutral ion beam with respect to the walls of the vacuum chamber and equilibrium temperature of plasma electrons are discussed. Then analysis of stability of the compensated cylindrical ion beam in region of drift is carried out. Electrostatic instability of the compensated ion beam in a drift tube is considered and the limiting current of quasineutral ion beam is obtained.